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Strategic Tools for the Formation of Cluster Capital to Implement Technological Innovations

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Abstract: In today's rapidly evolving digital landscape and accelerating technological development, industrial clusters play a crucial role in fostering innovation and ensuring sustainable economic growth. However, their effectiveness largely depends on the organization of optimal interactions between the participants, which implies a balanced allocation of resources and the co-evolution of capitals within the cluster. In this paper, we introduce strategic tools designed to form cluster capital by integrating financial, technological, and intellectual resources to create a sustainable environment for technological innovation implementation. To solve the set tasks, we developed a mathematical model based on the entropy approach and network analysis methods. This was developed to model and optimize the resource distribution among the cluster participants. The application of the proposed model using the example of the PenzaStankoMash industrial machine-building cluster has shown that the optimal configuration of the actors' capitals in clusters contributes to the creation of synergetic effects. This increases the innovation potential and overall efficiency of the system. Our modeling considered various capital allocation scenarios, leading us to conclude that a balanced approach is important. The results of this study contribute to an in-depth understanding of the mechanisms for optimizing interactions in clusters. They contain specific strategic tools for managing capitals in clusters and contribute to the development of industrial policy based on the principles of a systematic approach.

Keywords: cluster capital; technological innovations; synergy; entropy approach; resource optimization; industrial clusters; innovative ecosystems

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1. Introduction

In today's digital transformation and rapid technological development, industrial clusters are becoming important drivers of innovation and sustainable economic growth [1–3]. However, the effectiveness of their functioning largely depends on the optimization of the interactions in integration structures [4,5]. This allows for a balanced distribution of resources, the co-evolution of the capital elements within the cluster, and the maximization of synergistic effects. Researchers have identified the influence of capital co-evolution on the resource allocation efficiency within clusters, highlighting the importance of factors such as trust, training, and participation in innovation processes [6,7].

Cluster structures are complex networks that integrate enterprises, scientific organizations, infrastructure institutions, and other stakeholders. Their successful development requires a clear coordination of cooperation between actors aimed at achieving a common goal: increasing the competitiveness of each participant and achieving technological sovereignty.

One key aspect of managing cluster structures effectively is the principle of complementarity. This involves completing the necessary skills, resources, knowledge, and technologies to ensure the additionality of system elements. In other words, participants in a cluster should complement each other's resources and competencies, ensuring the harmonious development of the entire system. This approach allows each participant to contribute the resources that the others lack, creating a balanced ecosystem that benefits all parties involved.

Without an appropriate capital configuration, clusters face the following significant challenges in their development:

- The unequal allocation of resources among participants can reduce the overall efficiency and lead to system instability;
- The lack of integration of knowledge and technology can lead to the duplication of efforts and a decrease in innovation activity;
- Insufficient synergy between different types of capitals, such as technological, financial, and intellectual capitals, hinders the overall development of the cluster.

An optimal distribution of capitals among cluster actors can create a sustainable innovation environment where enterprises and organizations interact and form mutually beneficial co-evolutionary relationships. This allows for the success of one company to contribute to the growth of the whole system.

In today's digital age, as industrial systems become more complex, traditional business models face challenges related to data fragmentation and a lack of information integration. These challenges complicate analysis and decision making, especially for cluster structures that require flexible and adaptive capital management. Systems thinking offers an alternative approach by considering a cluster as an integrated system of interconnected elements. This allows us to identify patterns of interaction between internal and external factors and model optimal ways to redistribute resources. It also helps form strategies to maximize synergetic effects.

By adopting a systematic approach to cluster capital management, we can develop integration mechanisms that consolidate fragmented data, technologies, and skills into a single ecosystem. This improves the efficiency of technology investments and promotes better decision making within the cluster.

Cluster development requires an integrated approach to manage the various forms of capital—financial, technological, intellectual, social, institutional, and others. The optimal combination of these types of capitals enables the following:

- The creation of conditions for the integration of technological solutions and increasing their scalability;
- The development of collaboration mechanisms between cluster members to reduce transaction costs;
- The strengthening of the cluster's position in global markets through the collective innovation potential.

Thus, the configuration of these types of capitals and their balanced development within clusters is a strategic necessity that ensures not only the survival but also the sustainable growth of industrial systems. Technological capital is one of the key factors in achieving maximum synergy and efficiency. In the context of digital transformation and innovative development, technological capital is becoming the main engine of business processes within the cluster. It stimulates the creation of new products, services, and production processes. The effective integration and optimization of technological solutions within the cluster allow participants to enhance their competitiveness, innovate, and quickly adapt to market changes. Research shows that the co-evolution of technological capital with other types of capitals, such as financial and intellectual capitals, not only improves productivity and innovation activity but also enhances the interactions between participants, leading to the growth of the entire cluster [8–11]. In particular, the joint use of advanced technologies and resources allows cluster companies to reduce their transaction costs, increase their production capacities, and significantly accelerate the launch of new innovative solutions on the market.

This study is based on the hypothesis that the successful formation of cluster capital and its effectiveness depend on the network interaction of actors and the optimal allocation of resources. We propose the following research hypotheses:

H1. *The balanced development of cluster structures depends on the configuration of the financial, intellectual, innovation, and technology capitals within the cluster*;

H2. *Entropy modeling can be used to assess the uniformity of the capital distribution in industrial clusters.*

The present research aims to develop strategic tools for optimizing interactions and managing cluster capital. The research is organized in the following structure: Section 2 reviews the literature on the evolution of capital theory, the roles of technological and intellectual capitals in clusters, as well as the existing methodological approaches to optimizing them. Section 3 describes the methodological basis of this study, including mathematical modeling, network analysis, and an entropy approach. This approach is implemented to assess the co-evolution of cluster elements, ensuring internal balance and capital efficiency. In Section 4, we provide empirical research results using the example of the PenzaStanko-Mash industrial machine-building cluster. It demonstrates that the preferred setup of actors' capitals in clusters maximizes the cluster potential and increases the cluster innovation efficiency. We considered three scenarios to evaluate the effectiveness of the cluster capital optimization model and the impacts of various parameters on resource allocation. Section 5 provides strategic recommendations for government agencies and businesses regarding the development of innovative cluster systems. The results of this study will allow us to have a better grip on the mechanisms for optimizing interactions within clusters and to propose effective strategies for capital management in these environments. They also contribute to the development of industrial policy based on the principles of a systematic approach.

2. Literature Review

In today's digital economy and competitive global landscape, capital is no longer a solely material or financial category. Modern research highlights its multidimensional nature, including intellectual, technological, and cluster capitals, which play a key role in the development of innovative ecosystems. This section explores the evolution of capital as a concept, examining its modern forms and their interaction in cluster development.

2.1. The Evolution of Capital's Concept: From a Traditional to Multidimensional Understanding

Capital, as an economic concept, has historically been defined as a collection of assets capable of generating income. V.V. Radaev [12] identifies five key characteristics of capital: its limitation, ability to preserve and increase, possibility of conversion into monetary form, dynamism, and self-increasing value. However, economic development in modern conditions is increasingly dependent on intangible resources, such as knowledge, information, skills, and innovative technologies, rather than physical assets.

The concept of multiple capitals suggests the classification of capital into financial, industrial, intellectual, human, social, and natural categories [13]. This confirms the need for a systematic approach to capital management that considers the interrelationship of different types of resources.

The development of digital technologies has resulted in the emergence of new forms of capital. In particular, digital capital is a separate category influencing business models and economic strategies [14]. Although digital capital plays a crucial role in modern economies, its impact on cluster structures requires further study. Technological capital is more relevant for industrial development. It ensures the integration of innovative solutions into production processes.

2.2. Intellectual and Social Capitals and Their Roles in Cluster Development

Intellectual capital is a key element of the competitiveness of modern enterprises and clusters. According to D. Lanskaya et al., it includes knowledge, skills, patents, and innovative developments which form the basis of the value propositions of companies [15].

However, intellectual capital does not operate in isolation—its effectiveness is enhanced through its integration with technological and financial resources. The company's capital structure includes the "core", which comprises material, financial, human, and technological capitals, as well as the "fringe", which consists of virtual capital. The success of a business depends on implementing strategies at all levels, requiring the use of various tools, such as management, innovative projects, and corporate culture. A balanced distribution of intellectual capital within clusters is emphasized in the model of capitalograms, highlighting the importance of combining tangible and intangible assets for sustainable organizational growth [16].

In addition to the coordination of intellectual capital, it is essential to create effective mechanisms for knowledge exchange among the members of a cluster. The study [17] focuses on how intellectual capital can be measured and managed within cluster structures.

The authors of [18,19] also emphasize the importance and influence of intellectual capital on firms' productivity and entrepreneurial success in high-tech clusters.

Ben Zammel and Najar argue that successful clusters utilize collective knowledge management techniques, allowing for the distribution of intellectual capital among participants [20]. This not only allows for the accumulation of intellectual capital but also its dynamic redistribution. It creates conditions for innovative co-evolution among participants.

Furthermore, it is essential to consider social capital, which reduces transaction costs and enhances the cluster's innovative potential. Granovetter emphasizes the significance of the social embeddedness of economic activities, contributing to the formation of stable connections and trust among cluster members [21].

The formation of cluster capital requires the systematic management of social relations to strengthen knowledge transfer processes and increase the competitiveness of the cluster participants. A recent study by Cardoso et al. [22] demonstrates that social capital plays a crucial role in facilitating knowledge transfer and creating sustainable connections between companies in technology clusters. Using the example of two technology clusters, the authors show that the success of spin-off companies is primarily based on the quality of the networking and the integration of knowledge between the parent company and startups. Social capital is considered an essential element of cluster capital in [23], as it contributes to reducing transaction costs and enhancing the innovative potential of a region.

2.3. Technological Capital as a Factor of Sustainable Cluster Development

Technological capital is a combination of knowledge, information systems, technologies, and R&D investments that contribute to the development of innovative business models. According to research by Grigoriev et al., it is defined as a set of tangible and intangible assets related to production and management, making it a key factor for sustainable growth [24]. The authors define technological capital as the sum of a company's tangible and intangible components. Tangible assets include fixed assets, while intangible assets include those related to production and management. They emphasize the importance of this concept in the innovative development of companies, as it leads to an increase in their overall value, with a greater emphasis on intangible assets.

An in-depth analysis of the structure of technological capital is presented in the work of Gao et al. They propose an analytical framework that includes four aspects: the factors of technological capital formation, the processes of its capitalization, the fundamental mechanisms of interaction, and the effects of its use at the corporate and national levels [25]. The essential difference between technological capital and other related concepts, such as technology and technological resources, is that technological capital can be integrated with other forms of capital and "invested" in production directly without passing through the market.

However, the availability of technological capital alone is not enough for success. It is important to optimize its distribution within clusters using network analysis and entropy modeling tools to evaluate knowledge and resource flows [26].

Thus, the theory of capital continues to develop, and the mechanisms for engaging and optimizing the use of various forms of capital in the production process remain significant.

2.4. Cluster Capital: Integrating Resources to Maximize the Synergetic Effect

Cluster capital refers to the combined resources and assets resulting from the collaboration of various participants. This integration gives actors an advantage over competitors and the potential for development. It also provides an effective strategy for implementing change. Cluster capital emphasizes the importance of both tangible and intangible assets in supporting the growth and success of a cluster.

Understanding models of the cluster lifecycle is important for the processes of cluster adaptation and development. M. Porter [27] proposed a model that describes the evolution of a cluster through stages such as its origin, growth, maturity, and decline. Due to the dynamic nature of clusters, it is significant to consider constant change and adapt capital management strategies.

Each cluster group has its own types of capitals (industrial clusters, the agro-industrial complex, tourism clusters, medical clusters, etc.). Cross-industry clusters integrate potential and resources from various industries.

The basic structure of cluster capital includes the following components:

1. Technological capital: the availability of technologies and innovations to increase competitiveness and efficiency;
2. Intellectual capital: the knowledge, skills, and experience of employees to contribute to innovation and improve productivity within the cluster;
3. Financial capital: access to funding and investment necessary for growth and development within the companies in the cluster.

The Challoumis study [28] emphasizes that technological innovations have a multiplier effect on economic growth, reducing transaction costs and increasing cluster productivity. However, the key factor is the optimal configuration of different types of capitals, since uneven resource allocation can lead to imbalances and reduced efficiency.

Cluster capital optimization should consider the following:

- ✓ The principle of the co-evolution of capital elements, where technological, intellectual, financial, and other types of capitals develop simultaneously;
- ✓ The principle of complementarity, which allows for the completion of missing resources and skills through cluster interaction;
- ✓ The use of mathematical models to determine the levels of uncertainty in capital allocation.

Therefore, cluster capital management requires a strategic approach that considers not only the accumulation but also the dynamic redistribution of resources within the system. The optimization of the interactions within integration structures is crucial for their effective functioning. For example, too few interactions can result in the insufficient exchange of complementary resources, while too many interactions can complicate coordination and increase costs. Various methods can be employed to solve this issue, depending on the available data and goals of the integration structure.

To solve this problem, various methods can be used, depending on the source data and the goals of the integration structures. Here is a brief overview of them:

1. Methods of network analysis. A network is a complex system composed of several interconnected subsystems that interact with each other. The performance of the entire system is dependent on the effectiveness of its individual parts. Network analysis can be used to analyze the structure of interactions between economic actors, such as participants in integration structures [29]. This involves assessing interaction patterns and selecting a system of indicators, collecting data, and analyzing individual subnetworks [30].

In the scientific literature, network interactions are subject to comprehensive and extensive analysis. N. E. Egorova's work, for example, focuses on developing efficiency indicators that allow for the quantification of network interaction development [31]. A. A. Afanasyeva and S. P. Kushch analyze the factors that influence the effectiveness of these interactions [32]. V.L. Simonov and K.B. Rybalko have developed a system of indicators for evaluating the efficiency of network interactions, which includes four stages: creating a model that reflects how the network characteristics influence the outcomes of participants' activities; identifying factors that affect the effectiveness of interactions at various stages of the network lifecycle; analyzing the relationship between these factors; determining an integrated indicator for the interaction effectiveness, and creating profiles for network participants [33].

Thus, it is essential to collect data about the network structure to effectively use network analysis methods. This includes information about the participants (agents) and their interactions, as well as about the nature, frequency, and intensity of these interactions. Additionally, it is crucial to identify the key performance indicators for each participant and for the network as a whole. This allows us to evaluate the contribution of each agent to the overall outcome.

However, a drawback of using these approaches is that choosing the evaluation indicators can be subjective and depend on the preferences of the researcher. This may result in an incomplete understanding of all the significant aspects of the networking and reduce the accuracy of the assessment. Therefore, in addition to using an indicator system, it can be beneficial to utilize graph methods, which offer a more structured approach to examining and optimizing network structures.

The interacting structure can be represented as a graph, $G = (V,E)$, where V is the set of nodes (agents), and E is the set of edges (interactions between agents). The goal is to find a subset of edges (E′ ⊆ E) that maximizes a certain objective utility function (U(E′)) while respecting the constraints on the minimum and maximum numbers of interactions for each node. The utility function measures the total value of all selected interactions. Each interaction has a weight, which reflects its significance or quality. Optimizing the network structure is then formulated as follows:

$$\begin{cases} U(E') = \sum_{e \in E'} w(e) \to max \\ d_{min}(v) \le d(v) \le d_{max}(v),\ \forall v \in V' \end{cases} \tag{1}$$

where $w(e)$ is the weight of the interaction (for example, its usefulness or cost) and the degree of the node (number of interactions), and $d_{min}(v)$ and $d_{max}(v)$ are the minimum and maximum allowable numbers of interactions for the given node (v).

The task is to identify the most significant interactions between the nodes in the network, which together bring the maximum benefit. To study real-world integrations, it is important to know which nodes (economic agents) are interconnected. It is essential to characterize these connections in terms of their significance or quality and to assign the weight to each interaction. These data need to determine the network structure and evaluate its effectiveness. It is also important to take into account limits on the number of interactions that each agent can participate in to balance their involvement in the network. Additionally, we need to determine the target utility function that will be maximized during the network optimization process. This utility function should take into account the overall importance of all the selected interactions;

2. Modeling and simulation. This approach involves creating models that simulate the interactions between economic systems within an integrated association. This allows for the modeling of a large number of relationships to optimize the functioning of these systems. Agent-based modeling is often used for this purpose, where participants in economic relations are considered as agents interacting with each other in a simulated environment. This enables conducting simulation experiments with different levels and types of interactions to achieve the desired balance. Thus, the use of agent-based modeling in combination with simulation experiments is a method for studying the dynamics of complex systems. It consists of autonomous agents interacting with each other, observing the collective effects of their behavior and interactions [34].

For example, in Reference [35], a simulation–optimization model is presented to optimize the interaction between high-tech companies and financial institutions with the goal of maximizing the market value of a company's capital. The article [36] shows that agent-based modeling can be effectively used in the analysis of dynamic adaptive systems, helping to study the diffusion of innovations and the flow of knowledge and information.

While simulation and agent-based models are effective tools for examining interactions within economic systems, there are several drawbacks to this approach. For instance, the outcomes can significantly depend on the initial conditions and parameters, making them sensitive to data changes. Additionally, successful modeling requires accurate data, which may not always be readily accessible.

Analyzing real-world systems using this toolkit requires detailed information about the participants' characteristics, such as their behavioral patterns, available resources, and strategies. It is also helpful to have data on the types and levels of interaction between participants, including the frequency and intensity. Additionally, information about the external environment can also be valuable;

3. Game-theoretical models. Game theory can be applied in various fields of economics, such as labor, macroeconomics, and other applied fields. Similar problems arise in these different fields, and game-theoretic models provide a powerful tool for analyzing interactions between economic agents. These models help to understand strategic behavior

and decision making, aiming to find effective coordination and cooperation in relationships. They pay special attention to cooperation [37].

Game-theoretic modeling has become a popular tool in economics for making decisions. However, there has not been enough research conducted to determine the accuracy and effectiveness of this approach [38]. One article demonstrates how game theory can optimize investment planning and forecasting, as well as effectively manage these processes [39]. Other studies have shown the use of game-theoretical models in analyzing transport markets and interactions between decision makers [40], and in responding to the demand for electricity in growing electricity markets [41].

One tool for optimizing interactions in integrations is cooperative game theory. This field of game theory focuses on analyzing and optimizing the interactions between players to form coalitions and achieve common goals along with the fair distribution of resources. The goal of cooperative games is to maximize the overall benefits for all players while maintaining a balance between their interests.

In cooperative games, the utility function of the coalition (U(C)) is defined for the coalition (C⊆V, where V is the set of all possible players) and measures the overall benefit from the cooperation of all players in the coalition. Formally, the coalition's utility function can be written as follows:

$$U(c) = \sum\nolimits_{v \in C} u(v, C), \tag{2}$$

where $u(v,C)$ is the utility of a player (v) in a coalition (C).

To determine the contribution of each player (v) to the overall utility of the coalition, the Shapley value is used. This value helps to balance the interactions, taking into account the individual contribution of each player [42]:

$$\phi(v) = \sum\nolimits_{C \subseteq V \setminus \{v\}} \frac{|C|! \cdot (|V| - |C| - 1)!}{|V|!} \cdot (U(C \cup \{v\} - U(C)) \tag{3}$$

where C is a subset of players, excluding v, and $U(C \cup \{v\}$–$U(C)$ is player v's contribution to the C coalition.

To effectively use game-theoretic models in analyzing real-world integration scenarios, it is crucial to have detailed information about the participants and their characteristics, resources, and strategies. Additionally, data on coalitions, conditions for their formation, and outcomes are essential, as well as knowledge of utility functions. It assesses the benefits of cooperation and how individual contributions influence the overall coalition benefit;

4. Models of random processes. Random process models, such as Markov chains, provide powerful tools for analyzing interactions between economic agents. These interactions can be described as a Markov process, where the probability of moving from one state to another depends on the current state of the system. And this reflects the current number of interactions. The goal of these models is to optimize the transition probabilities so that the desired state with the optimal number of interactions is achieved.

Markov models are especially useful for studying the interactions among many agents with asymmetric fluctuations, such as business cycles. They can also be used to describe systems with multiple locally stable equilibrium points, which depend on uncertainties [43]. For example, the hidden Markov model was used to study customer relationships and their effect on customer behavior in [44].

In such models, the transition probabilities, denoted as p_{ij}, play a key role, as they determine the likelihood of the system moving from state i to state j at time t:

$$P(X_{t+1} = j \mid X_t = i) = p_{ij} \tag{4}$$

Optimizing these transition probabilities allows us to either minimize or maximize the objective function related to the number of interactions in the system.

When analyzing real-world systems with models based on random processes, it is important to have a comprehensive understanding of the system's current states and the probabilities associated with the transitions between these states. Additionally, knowing the target function that we want to optimize is essential for making informed decisions about how to adjust the system's parameters.

Thus, modern research supports the need for a comprehensive approach to managing resources within clusters. To optimize interactions within integration structures, it is essential to effectively organize financial, technological, and intellectual capitals, and to effectively distribute these resources among participants. However, the challenge of developing mathematical modeling methods for optimizing capital configurations within cluster structures remains significant.

3. Materials and Methods

One of the most promising approaches to optimizing interactions between actors in integrated systems is based on the concept of entropy. The asymmetric distribution of resource flows in industrial clusters indicates the existence of economic entropy, which regulates these imbalanced processes [45]. The concept of economic entropy arises from the fact that flows (of knowledge, information, and capital) are analogous to energy in thermodynamics [46]. Translating these ideas into a system of economic coordinates, an increase in economic entropy within a cluster can be seen as an increase in uncertainty about the costs of capital, goods, and values. Uneven changes in the use of various types of capitals lead to a violation of the principle of capital constancy and a shift in the equilibrium position. This can result in an unstable state for the cluster. Additionally, the industrial cluster is a complex and open system that transfers energy, in the form of information, knowledge, and skills, to the external environment. The sustainable development of the cluster depends on a balanced distribution of resources among the actors.

The choice of the entropy approach was made due to its ability to analyze the uncertainty and uniformity of resource allocation in complex systems, such as industrial clusters. Unlike dynamic game theory, which focuses on the strategic behavior of rational agents [37], the entropy model allows us to quantify the balance between efficiency and stability, which is crucial for clusters with diverse participants. Entropy methods have been successfully applied in economic research. For example, Islamutdinov and Kurikov [46] demonstrated their effectiveness at assessing the stability of regional systems. Unlike econometric models, which rely on strict assumptions about linearity in data, entropy approaches are flexible and adaptable to incomplete data sets. This makes them preferable for analyzing clusters that have a high degree of uncertainty. In addition, entropy allows for the integration of network analysis, identifying key actors and their influence on the system, which is not possible with classical regression approaches.

To formalize this approach to solving the problem of optimizing interactions between economic agents, we can consider a system consisting of N economic agents. Each agent (i) has a set of actions (Ai) and the probability (pi(a)) of performing an action (a∈Ai). The utility of each agent is represented by a function, Ui(x), where x is a vector of the resources distributed among the agents. The usefulness of an agent may depend on the allocation of resources and the actions of other agents.

The entropy of the system measures the uncertainty in the resource allocation, or the strategies chosen by agents. We use Shannon entropy to define the probabilistic distribution of agent actions [47]:

$$(p) = -\sum_{a \in A} p(a) \log p(a), \tag{5}$$

where $p(a)$ is the probability that a certain action (a) will be chosen.

Where the distribution of resources (x) between agents is considered, multidimensional entropy can be used to estimate the uncertainty of the resource distribution:

$$H(x) = -\sum_{x \in x} p(x) \log p(x), \tag{6}$$

where $p(x)$ is the probability of resource allocation (x).

Then, the optimization problem is formulated as follows: finding a distribution of resources and actions that maximizes a utility function subject to entropy constraints.

Let $U(x)$ denote some function of the integration efficiency, which depends on the resource allocation (x). The optimization can be written as follows:

$$\max_{x,p} [U(x) - \lambda H(x)], \tag{7}$$

where λ is the coefficient that determines the significance of minimizing entropy (or uncertainty). This parameter controls the weight of the entropy minimization within the overall optimization problem. The large value of λ makes minimizing entropy a more important factor than maximizing the utility function ($U(x)$).

In Expression (3), the entropy ($H(x)$) measures the uncertainty or uniformity of the resource allocation. A higher entropy value indicates that resources may be distributed more randomly, while a lower entropy may indicate a more specific distribution.

To optimize the cluster's capital, the authors developed a mathematical model that considers the specific functioning of each actor in the industrial cluster. This model ensures an even distribution of resources and identifies the contribution of each type of capital to the overall outcome.

The model's variables and parameters are as follows:

- K_C is the cluster capital (the objective function to maximize);
- F_i is the financial capital of actor i;
- T_i is the innovative and technological capital of actor i;
- I_i is the intellectual capital of actor i;
- x_{ij} is the share of resources allocated to the capital (j) of actor i (a controlled variable);
- w_{ij} is the weight (importance) of the capital (j) for actor i (an expert-defined parameter).

To confirm the weighting coefficients and the significance of the various forms of capital in the model, a survey was conducted using the Delphi method. The study involved 15 experts specialized in cluster management and innovation from the Penza region. The experts included representatives from the Cluster Development Center, relevant ministries, government agencies, and other departments. During several rounds of the survey, the experts agreed on the estimates, which were then statistically verified for consistency (with the Kendall's concordance coefficient (W) = 0.78 and $p < 0.05$). The expert assessments fully supported the weight coefficients suggested in the model.

The total cluster capital is calculated by taking the weighted sum of all the actors' capital types, considering their resource allocations:

$$K_C = \sum_{i=1}^{n} \sum_{j \in \{F,T,I\}} x_{ij} \cdot w_{ij} \cdot K_{ij}, \tag{8}$$

where K_{ij} is the normalized value of the capital (j) of the actor (i).

To account for the uniformity of the resource allocation, the entropy functional is introduced:

$$\mathrm{H} = -\sum_{i=1}^{n} \sum_{j \in \{F,T,I\}} x_{ij} \cdot \ln(x_{ij}), \tag{9}$$

where H measures the degree of uniformity in the distribution.

The adjusted objective function then becomes the following:

$$K_C = \sum_{i=1}^{n} \sum_{j \in \{F,T,I\}} x_{ij} \cdot w_{ij} \cdot K_{ij} + \lambda \cdot H, \quad (10)$$

where λ is the ratio of importance given to the uniformity, as determined by the expert.

We also impose the following constraints on the model:

(1) Total resource allocation:

$$\sum_{i=1}^{n} \sum_{j \in \{F,T,I\}} x_{ij} = 1. \quad (11)$$

(2) Boundaries of x_{ij} variables:

$$0 \le x_{ij} \le 1, \ \forall i, \ \forall j \quad (12)$$

To compare indicators between the actors, we normalize the values using the minimax method:

$$K_{ij}^{norm} = \frac{K_{ij} - K_j^{min}}{K_j^{max} - K_j^{min}} \quad (13)$$

where K_{ij} is the value of the capital (j) of actor i; K_j^{min} and K_j^{max} are the minimum and maximum values for the capital (j) across all actors.

The constraints of the model may include resource limitations, such as ensuring that the number of resources allocated is equal to the total available resources. Additionally, there may be restrictions on the actions that actors can take during interactions. To solve the optimization problem, mathematical methods, such as the gradient descent algorithm, can be used. The model calculations were performed using the scipy.optimize module, which provides solutions for nonlinear optimization problems with specified constraints and limits. Specifically, the Sequential Least-Squares Programming (SLSQP) optimization method was employed for tasks involving linear constraints and range variables.

Let us present an extended algorithm for the modeling process:

- Data collection: assessment of financial, technological and intellectual capital of actors.
- Normalization: reduction of indicators to a dimensionless form using the minimax method.
- Expert weighting: determination of weighting factors for each type of capital.
- Model formulation: integration of entropy into the objective function.
- Optimization: solving the problem using the SLSQP method while considering constraints.
- Validation: analysis of scenarios and sensitivity of parameters.

4. Results

To test the proposed mathematical model for optimizing the capital of the cluster in the interaction between actors, we considered the innovative industrial machine tool cluster based on OOO StankoMashStroy. In 2024, due to reorganization, the PenzaStankoMash innovation cluster gained the status of an interregional cluster. The cluster consists of 16 companies located in the Vladimir, Ryazan, Samara, and Penza regions, as well as educational institutions and infrastructure organizations. PenzaStankoMash is an innovative cluster specializing in the production of high-tech metalworking equipment with its own foundry [48]. Table 1 provides information on the key actors and their roles within the cluster.

Table 1. Composition of the main actors and their functions in the PenzaStankoMash cluster.

Actor	Functional Role in the Cluster
OOO PenzMetallProm	It supplies metal products, such as metal blanks and parts necessary for the assembly of equipment.
OOO Promservis	It provides equipment maintenance, including repair and commissioning, as well as the supply of spare parts.
OOO Ruselprom. Electrical machines	It manufactures and supplies electric motors and related components for CNC lathes.
OOO MashProm	It specializes in the supply of complex mechanical components, such as headstocks, skids, and covers for equipment.
AO TekhnoPortal	It supplies spindles for metalworking machines.
OOO Lazer PRO	It provides systems for cleaning cutting fluids (CFs) and laser complexes for material processing.
Krasny Oktyabr Corporation	It supplies high-quality rolled metal products used in the manufacture of machine housings and parts.
OOO Stanko-Lit	It manufactures cast iron castings for housings, guides, and other key elements of machine tools.
ZAO Vologda Bearing Corporation	It provides the supply of bearings for equipment, including components with high-precision operation.

The cluster includes suppliers of materials, components, and services that support the production process. Each company either supplies materials and components or provides maintenance services. OOO StankoMashStroy is an integrator in the metalworking equipment production cluster. The company specializes in high-quality metalworking equipment.

Capital optimization was conducted using a mathematical model that involved key production actors, including principal production companies. The model used three indicators:

(1) Financial capital (return on invested capital—ROIC), reflecting the efficiency of the invested capital use;

(2) Innovation and technology capital, assessing a company's technological potential;

(3) Intellectual capital (value-added intellectual coefficient—VAIC), measuring a company's ability to create value through human, structural, and innovation capital.

At the first stage, all values were normalized using the minimax method (Formula (9)) to correctly compare the indicators (financial, innovation and technology, and intellectual capitals) of different actors.

The normalized results are presented in Table 2.

Table 2. Normalized data on financial, innovation and technology, and intellectual capital indicators of actors.

Actor	Financial Capital	Innovation and Technology Capital	Intellectual Capital
OOO StankoMashStroy	0.2292	0.871	1.0000
OOO Mekhatronika	0.3505	0.000	0.0588
OOO Lazer PRO	0.3983	0.065	0.0588
OOO Inmash Concern	0.2805	0.516	0.0384
ZAO Vologda Bearing Corporation	0.2429	0.387	0.0000
Krasny Oktyabr Corporation	1.0000	1.000	0.0557
OOO Stanko-Lit	0.0000	0.065	0.1411

At the second stage of the modeling process, weights were assigned and the coefficient λ was determined. These weights were determined for each type of capital using the expert method. The factors considered included the following:

- Financial capital is important for companies with high ROICs;
- Companies with high technological potential received increased weights for innovation and technology capital;
- The importance of intellectual capital depended on its potential to create added value.

The distribution of the weighting coefficients is shown in Table 3.

Table 3. Distribution of weighting coefficients.

Actor	w_F	w_T	w_I
OOO StankoMashStroy	0.4	0.4	0.2
OOO Mekhatronika	0.3	0.3	0.4
OOO Lazer PRO	0.3	0.3	0.4
OOO Inmash Concern	0.4	0.3	0.3
ZAO Vologda Corporation	0.4	0.4	0.2
Krasny Oktyabr Corporation	0.5	0.4	0.1
OOO Stanko-Lit	0.3	0.4	0.3

Table 4 compares the initial expert estimates with the coefficients adjusted based on the results of the Delphi method. The data show that the initial and refined estimates for most parameters are similar or close, which confirms the correctness of the initial model structure.

Table 4. Comparison of initial expert estimates and adjusted coefficients obtained by the Delphi method.

Actor	Type of Capital	Initial Coefficient (Expert Assessment)	Adjusted Coefficient (Delphi Method)	Difference
OOO StankoMashStroy	Financial Technological Intellectual	0.40 0.40 0.20	0.38 0.42 0.20	−0.02 +0.02 0.00
OOO Mekhatronika	Financial Technological Intellectual	0.30 0.30 0.40	0.28 0.31 0.41	−0.02 +0.01 +0.01
OOO Lazer PRO	Financial Technological Intellectual	0.30 0.30 0.40	0.29 0.30 0.41	−0.01 0.00 +0.01
OOO Inmash Concern	Financial Technological Intellectual	0.40 0.30 0.30	0.39 0.32 0.29	−0.01 +0.02 −0.01
ZAO Vologda Bearing Corporation	Financial Technological Intellectual	0.40 0.40 0.20	0.40 0.41 0.19	0.00 +0.01 −0.01
Krasny Oktyabr Corporation	Financial Technological Intellectual	0.50 0.40 0.10	0.49 0.41 0.10	−0.01 +0.01 0.00

Table 4. *Cont.*

Actor	Type of Capital	Initial Coefficient (Expert Assessment)	Adjusted Coefficient (Delphi Method)	Difference
OOO Stanko-Lit	Financial	0.30	0.31	+0.01
	Technological	0.40	0.39	−0.01
	Intellectual	0.30	0.30	0.00

Note: The difference is calculated as "the Delphi Coefficient minus the Initial Coefficient". A positive value indicates that the significance of the capital's type has increased, while a negative value indicates the opposite. The results are based on expert approval.

Table 4 presents the results of the initial expert assessments and the adjusted coefficients obtained after applying the Delphi method. The data analysis reveals a slight difference between the initial and adjusted coefficients, not exceeding ± 0.02. This indicates a high level of consistency between the initial expert opinions and the final estimates. It confirms the correctness of the selected weighting coefficients and the validity of the proposed methodology for optimizing cluster resource allocation.

The results of the resource allocation optimization process indicate which actors within the cluster receive more or less resources. It is based on their financial, innovation and technology, and intellectual capital values, as well as the significance of these values (weights) for each actor.

At the third stage, we examined three scenarios to evaluate the effectiveness of the cluster capital optimization model and assess the impacts of different parameters on the resource distribution. For each scenario, we modified key model parameters, such as the uniformity coefficient (λ) and capital weights (w_{ij}), allowing us to assess how changes to the model's variables affected the allocation of resources among actors.

4.1. Scenario 1. Basic Version

For the initial testing of the model, we used the following parameters:

(1) The normalized capital values (shown in Table 2);

(2) The weighting coefficients of capital determined by the expert method, considering the profiles of companies (presented in Table 3);

(3) The coefficient $\lambda = 0.5$.

Based on these values, we calculated the optimal resource allocation, as shown in Table 5.

Table 5. Optimal allocation for the first scenario.

Company	Financial Capital (x_F)	Innovation and Technology Capital (x_T)	Intellectual Capital (x_I)
OOO StankoMashStroy, Penza, Russia	0.0434	0.0726	0.0540
OOO Mekhatronika, Ivanovo, Russia	0.0445	0.0362	0.0379
OOO Lazer PRO, Penza, Russia	0.0458	0.0376	0.0379
OOO Inmash Concern, Sterlitamak, Russia	0.0452	0.0492	0.0371
ZAO Vologda Bearing Corporation, Vologda, Russia	0.0438	0.0492	0.0362
Krasny Oktyabr Corporation, Volgograd, Russia	0.0984	0.0805	0.0366
OOO Stanko-Lit, Serdobsk, Russia	0.0362	0.0381	0.0394

The resource allocation results demonstrate that the model considers the significance of various types of capitals for each company. OOO StankoMashStroy receives the largest share of resources for innovation and technology, as its focus is on technological development. At the same time, a significant amount of attention is paid to financial capital. OOO

Mekhatronika and OOO Lazer PRO receive relatively small shares of all types of capitals, with a focus on financial capital. This indicates the need to strengthen their financial position before making further investments. OOO Inmash Concern and ZAO Vologda Bearing Corporation stand out with relatively high shares of technological capital, indicating their strategic goal of increasing their technological potential. The Krasny Oktyabr Corporation receives a large share of financial capital and has a significant focus on innovation and technology capital. These facts confirm its leading position in the cluster and the need to maintain its current competitive advantages. OOO Stanko-Lit has a more balanced distribution of resources with a slight emphasis on intellectual capital, possibly due to the company's desire to increase its innovation potential for future growth.

Therefore, each actor can use these results to focus on key areas and maximize the total capital, amounting to 1.6599.

4.2. Scenario 2. Reduction in λ

For the second test of the model, the following parameters were used:

(1) Normalized capital values (presented in Table 2);

(2) Capital weights determined by an expert method based on company profiles (shown previously in Table 3);

(3) The coefficient $\lambda = 0.1$.

This resulted in the optimal resource allocation presented in Table 6.

Table 6. Optimal capital allocation for the second scenario.

Actor	Financial Capital (x_F)	Innovation and Technology Capital (x_T)	Intellectual Capital (x_I)
OOO StankoMashStroy	0.0090	0.1171	0.0266
OOO Mekhatronika	0.0102	0.0036	0.0045
OOO Lazer PRO	0.0119	0.0043	0.0045
OOO Inmash Concern	0.0111	0.0169	0.0041
ZAO Vologda Bearing Corporation	0.0094	0.0169	0.0036
Krasny Oktyabr Corporation	0.5321	0.1965	0.0038
OOO Stanko-Lit	0.0036	0.0047	0.0055

The decrease in λ has shifted the focus towards resource concentration. Krasny Oktyabr Corporation receives more than 70% of all resources, with 53.2% going to financial capital and 19.7% to innovation and technology capital. This confirms its leading role in the cluster. The optimized cluster capital has decreased significantly, amounting to 0.5629, indicating a significant reduction in uniformity.

4.3. Scenario 3. Increasing the Weight of Intellectual Capital

For the third test of the model, the following parameters were used:

(1) Normalized capital values (as presented in Table 2);

(2) An increase in the weight of intellectual capital by 40%, with the weights for financial capital and innovation and technology capital reduced by 20% each;

(3) The coefficient $\lambda = 0.5$.

This resulted in the optimal resource allocation presented in Table 7.

Increasing the weight of intellectual capital allows for the redistribution of resources to support actors with high innovation potential. OOO StankoMashStroy and OOO Stanko-Lit have seen significant increases in their shares of intellectual capital resources, with 6.58%

and 4.25%, respectively. The optimized cluster capital amounts to 1.6413, demonstrating balanced growth.

Table 7. Optimal capital allocation for the third scenario.

Actor	Financial Capital (x_F)	Innovation and Technology Capital (x_T)	Intellectual Capital (x_I)
OOO StankoMashStroy	0.0442	0.0703	0.0658
OOO Mekhatronika	0.0444	0.0376	0.0399
OOO Lazer PRO	0.0454	0.0387	0.0399
OOO Inmash Concern	0.0448	0.0480	0.0388
ZAO Vologda Bearing Corporation	0.0438	0.0480	0.0376
Krasny Oktyabr Corporation	0.0836	0.0712	0.0387
OOO Stanko-Lit	0.0376	0.0391	0.0425

Figure 1 illustrates the relationship between optimized cluster capital and the parameter λ. As λ increases, the cluster capital grows linearly, reflecting the positive impact of uniformity on the system efficiency. λ balances the trade-off between maximizing the overall cluster capital and ensuring balanced resource allocation among actors. However, an excessive value of λ (e.g., 1) is impractical in real-world economics, as it implies the absolute priority of uniformity over efficiency. This is not consistent with the fact that companies require different levels of capital based on their specific needs. To strike a reasonable balance, we recommend that the λ values range from 0.2 to 0.4, allowing for both uniformity and flexibility in resource distribution. This helps maximize the overall capital of the cluster and maintain its dynamic growth.

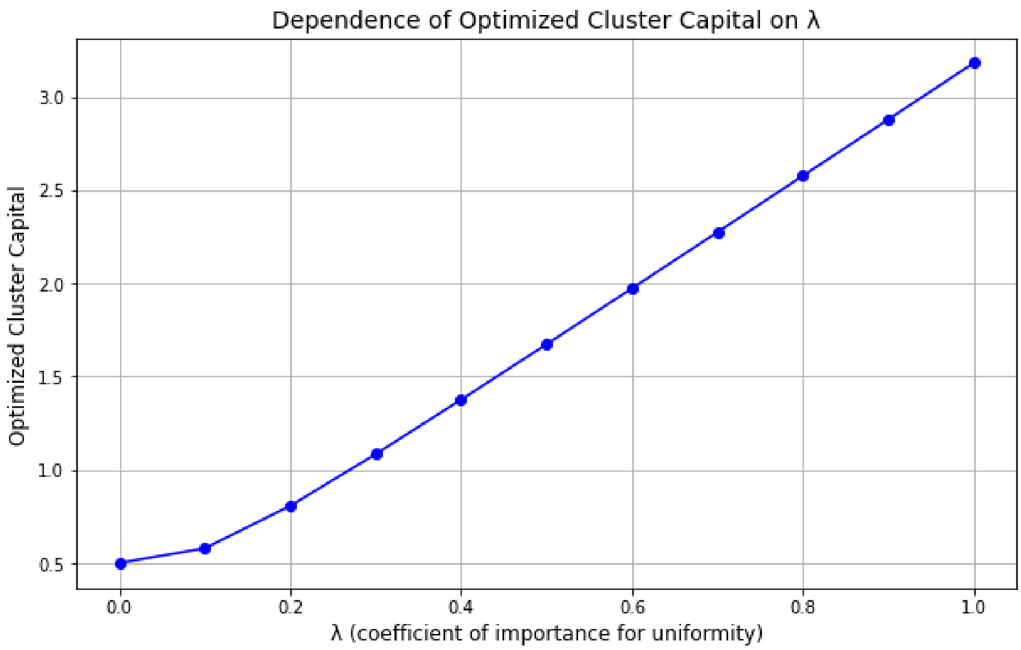

Figure 1. The dependence of the optimized cluster capital on the λ parameter. Source: data compiled by the authors.

Figure 2 shows a three-dimensional visualization of the changes in the cluster capital. It depends on the variation in the weighting coefficients w_1 and w_2 at a fixed value of $\lambda = 0.3$.

The graph illustrates the influence of various combinations of weights on the cluster capital value. The optimal combination of weights, where the maximum cluster capital value is achieved, is $w_1 = 0.1, w_2 = 0.9, w_3 = 0.0$. The absence of an intellectual capital component in this optimal combination does not mean that intellectual capital is unimportant. This value may reflect the specific characteristics of the particular cluster, or it could indicate that, for the given strategic goals and current circumstances, the contribution of intellectual capital to the cluster capital was insignificant. It also does not rule out the need to consider intellectual capital in other situations or by other actors where its role might be more significant or even crucial.

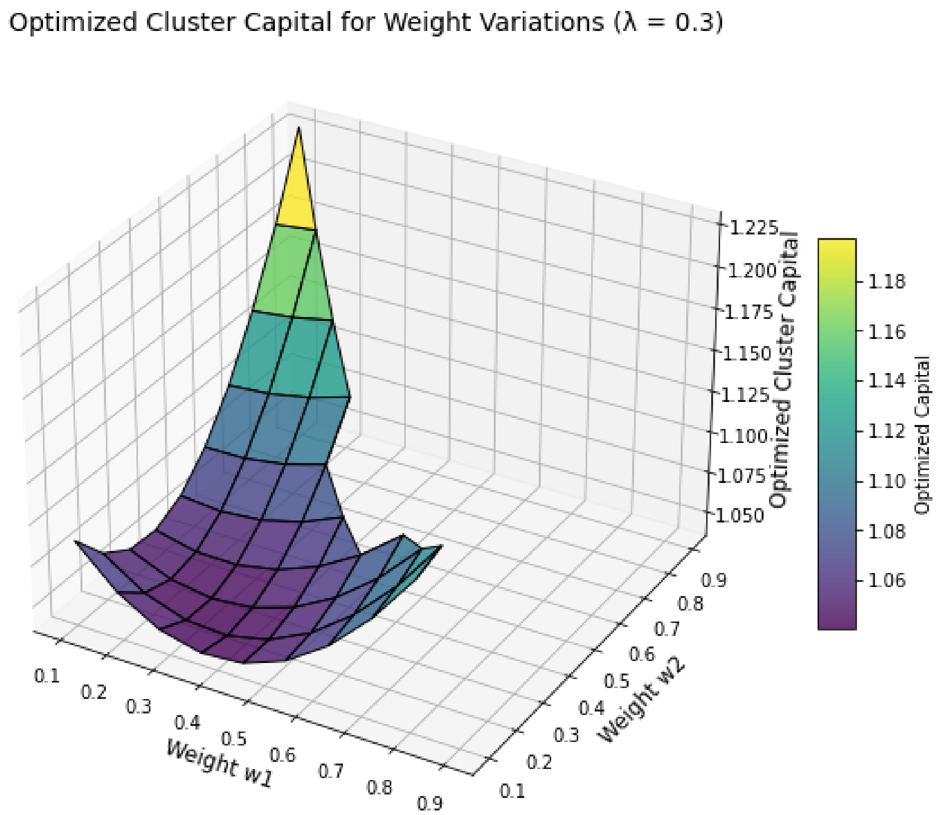

Figure 2. Three-dimensional visualization of changes in cluster capital. Source: data compiled by the authors.

A sensitivity analysis was conducted to confirm the accuracy of the model and evaluate the impacts of significant variables on resource allocation. The analysis of the function's structure (10) and Figure 1 reveals that the effect of λ on the final cluster capital (K_C) is linear. This pattern can be explained by the addition of the entropy term $\lambda \cdot H$, which proportionately increases the total capital while meeting constraints (11) and (12). Therefore, the sensitivity of the model to this variable does not reveal any critical points, and its selection is determined by practical economic factors. In addition to examining the influence of λ, we also performed a sensitivity test on the model for changes in the capital weights w_F, w_T, and w_I.

The sensitivity analysis algorithm is as follows:

1. Weight variation. For each parameter (w_F, w_T, w_I), the weight was altered by $\pm 20\%$, while maintaining the sum of weights equal to one.
2. Model recalculation. For each change, the optimization was performed using the SLSQP method, considering the imposed restrictions.
3. Results comparison. The change in the final capital of the cluster K_C was estimated.

The analysis results indicate that intellectual capital (w_I) has the most significant impact on the KCK (+/−18%), while financial and technological capitals have moderate effects (+/−7% and +/−9%, respectively).

The sensitivity analysis results showed that the model demonstrated resilience to changes in the key parameters. The results remained stable and maintained the overall trend of capital growth and balanced resource allocation. The model is not subject to critical fluctuations and can adjust to variations without significant deviations. This is confirmed by the linear relationship between λ and the cluster capital. It is also connected to the moderate impact of changes in the capital weights (particularly intellectual capital) on the final result.

4.4. Strategic Tools for the Development of the PenzaStankoMash Innovation Cluster

According to the results of the assessment, OOO StankoMashStroy from the PenzaStankoMash cluster in the first scenario (basic version) has the highest value of innovation and technology and intellectual capitals, with a shortage of financial capital. Krasny Oktyabr Corporation has a high proportion of financial and innovation and technology capitals. The balance of capitals in these two companies can be achieved by redistributing the financial flows between them. This will ensure a reduction in the innovation cycle and in the cost of innovation activities. All other members of the cluster have balanced capitals. For the first scenario, it is proposed that the PenzaStankoMash cluster follow an innovation development strategy based on its current innovation potential and balanced capital. To implement this strategy, the establishment of an infrastructure platform for cluster cooperation based on the principles of a project office is recommended. In the future, as the cluster grows, such a platform could become cross-industry and regional, allowing for effective information exchange between all stakeholders.

When implementing the second scenario (with a low level of uniformity in the distribution of cluster capital), the adoption of a cluster stabilization strategy is recommended. This strategy should include the use of two strategic tools. The first one is cluster control, used to increase transparency and trust among cluster members. And the second one is a cluster fund for mutual financial support among cluster members (an "overflow" of capital within the cluster). Cluster members have competitive advantages in the form of specialization. This specialization makes it possible to minimize the cost of introducing innovations due to secondary effects. They are manifested in the creation of innovative products and services at the border of different fields of knowledge. Consequently, the strategic development of the cluster is based on self-organization mechanisms through the cross-evaluation of each other's contributions by cluster participants at each stage of the innovation project implementation. According to the results of the assessment, it is necessary to "overflow" capital. This helps to ensure a balanced allocation of resources, the co-evolution of the cluster capital elements, the maximization of synergetic effects, and improvement in the competitiveness of each participant in the industrial cluster.

There is a balanced distribution of cluster capital in the third scenario. It would be reasonable to implement a strategy to maintain the economic performance. This strategy would involve the use of various tools, such as creating a cluster environment, which is a network of interactions. The environment is supported by regulatory and coordinating bodies at both the federal and regional levels. The best outcome in this environment can be achieved through synergies and agglomeration effects resulting from the optimal allocation of resources. A cluster environment is an integral part of the broader innovation, education, public administration, and private production landscape. The executive authorities of the constituent entities of Russia and local governments play a significant role in initiating cluster development and providing subsidies for measures to support their organizational development. Targeted investments are aimed at the development of engineering, transport infrastructure, and housing construction considering the goals of cluster development.

When analyzing and comparing different strategic instruments for creating cluster capital, it is important to prioritize options that promote the stability of clusters. We can achieve this by synthesizing object-, environmental-, process-, and project-based strategies. The inherent characteristics of objects and processes should balance the instability inherent in individual designs and environmental systems. A key goal of cluster strategic planning is to maintain a balance between the three main types of capitals, financial, innovation and technology, and intellectual capitals, for the cluster, as well as for individual participants. The interrelation of many cluster capitals makes a significant shift in one type improbable without similar changes in the others. Intellectual capital forms the basis for innovation and technology capital. Environmental factors influence financial capital, which affects both intellectual and innovation and technology capitals. Furthermore, we should synchronize the cluster capital formation strategy with those of the cluster participants, considering their different lifecycle stages. We can only achieve this by using tools such as multilevel strategic planning and gradually integrating participants' capitals into the cluster system. This aims to optimize the allocation and effective use of cluster capital.

5. Discussion and Conclusions

In today's digitalized and innovative world, industrial clusters are crucial for achieving sustainable economic growth and technological independence. One of the key factors for their success is the effective management of cluster capital. This involves a balanced interaction between financial, technological, and intellectual resources. This article continues a series of studies by the authors aimed at improving the efficiency of cluster systems [49–53]. Within this study, we have developed a methodological approach to optimize cluster capital using a mathematical model based on entropy and network analysis. This allows us to evaluate and allocate resources in a cluster to maximize innovation and competitiveness. Network analysis helps identify the central actors within the cluster and their impacts on the capital distribution. This was confirmed by studies conducted by Borgatti et al. [54]. They used network analysis to identify key participants in cluster networks. Newman also examines the structures of interactions in economic and social networks [55]. The entropy approach proposed by Shannon [56] and adapted in the economic models of Jakimowicz [57] helps to analyze the levels of uncertainty in the allocation of capital. This allows for more precise adjustments to the allocation of resources within the cluster.

The application of the proposed model, using the example of the PenzaStankoMash industrial machine tool cluster, has shown one important thing: an optimal allocation of resources among cluster members contributes to the creation of synergistic effects, improvements in the innovation potential, and the enhancement of the overall system efficiency. As a result of the modeling process, various scenarios for capital allocation were analyzed. This led to the conclusion that a balanced approach to the allocation of all types of capitals is essential for ensuring sustainable growth and innovation within the cluster.

The proposed cluster capital model aligns with several existing approaches in cluster research. For instance, studies by Tsvetkov and Stepnov [58,59] discuss hybrid clustering models that incorporate cooperation between participants in various stages of business processes. This is similar to our model's emphasis on efficient capital allocation among actors to foster synergies and enhance interconnections within the cluster. Additionally, there is a resemblance to approaches that advocate for the separate accounting of costs and capital flows within clusters, crucial for accurately assessing economic efficiency and optimizing resources.

In addition, this approach aligns with the theory of capital co-evolution described by Kleiner and Kvint [60,61]. They emphasize the significance of consistency and complementarity between different types of capitals in creating sustainable and efficient cluster systems. This also enhances industrial integration.

The approaches proposed by Jackson [62] and Goyal and Vega-Redondo [63] focus on investigating interaction strategies within cluster systems. Unlike them, this model incorporates additional elements of uncertainty and network analysis to more accurately optimize cluster capital.

The developed mathematical model for optimizing cluster capital has shown its effectiveness through testing, using the PenzaStankoMash machine tool cluster as an example. This model is based on an entropy approach and network analysis, but there are many aspects that require further development. For example, we need to consider external factors such as government regulations, market fluctuations, and geopolitical risks. These factors could enhance the model's resilience to real-world economic conditions. An analysis of how sensitive these variables are would allow us to see how they affect resource allocation and cluster adaptability. However, integrating these parameters requires detailed environmental data, which are often unavailable due to their dichotomy, heterogeneity, or lack of transparency. We focused this study on the cluster's internal interactions because of the specifics of the accessible information and the need to optimize the capital co-evolution.

Moreover, we could improve the model's ability to generalize by testing it on groups of other industries. However, extending the sample size is hindered by challenges in collecting comparable data. This is especially difficult due to differences in reporting methods and information confidentiality. Additionally, the transition from expert-determined weights based on historical data requires a significant amount of information. But it is currently unavailable in open statistics. Despite their subjectivity, expert assessments remain a necessary tool for a limited empirical basis.

Another promising area is the development of a dynamic version of the model that considers the gradual evolution of the cluster. Dynamic modeling, however, requires long-term data on capital changes, interactions, and external conditions. And they are also rarely available in sufficient volume. This work used a static approach as a methodological basis; future additions of temporal components will depend on the accumulation of relevant data.

The proposed approach shows promising results. However, there are limitations to consider when applying it in real-world scenarios. Firstly, the approach relies on accurate and up-to-date data about the capitals of different actors and their interactions. This can be challenging in situations where information is limited, especially for intangible assets. Secondly, the determination of the weighting coefficients for different types of capitals can be subjective. This can lead to potential errors in the results because the values of these coefficients depend on the specific characteristics of each cluster. Finally, the proposed model focuses on a static allocation of resources and may not account for the dynamic nature of changes in the environment, such as technological advances or economic crises. It is also important to note that the model does not consider social and institutional factors, such as trust between participants and institutional constraints [64], which can significantly affect the effectiveness of interactions.

Therefore, Powell et al. [65] and Chen et al. [19] consider these factors to be crucial for the successful operation of clusters. This is especially significant in areas with intensive capital and knowledge exchange between actors, such as biotech companies and venture capital firms. In the future, the authors plan to supplement their quantitative model with qualitative metrics. They plan to use surveys and expert interviews to measure trust, cooperation incentives, and institutional support among participants. The authors will also consider the quality and level of institutional support. This will ensure an integrated approach and a comprehensive analysis of the success factors of clusters.

Future research will focus on applying machine learning techniques to predict the dynamics of cluster capital and analyze interactions among cluster members using big data. Model development may involve the use of multi-factor econometric models that incorporate socio-economic variables. This will improve the accuracy of predictions regarding cluster capital dynamics. This aligns with Ladynin's research [66], where the use of such models has demonstrated high effectiveness at predicting economic security at the level of the cluster system. Additionally, it is crucial to develop new models to more precisely assess the impact of technological advancements on cluster capital in light of the rapid development of AI technologies.

In future research, we should focus on creating a platform based on the project office principle for cluster capital management. This platform could facilitate more effective communication among cluster participants, ensuring the coordination of projects and resources, as well as simplifying the decision-making process and strategy implementation. Additionally, it is essential to investigate the use of capitalograms. They can visualize and systematize the relationships between various types of capitals. This can provide a clear picture of the current state and dynamics of the cluster's capital. This will help all participants understand the composition and trends of the cluster, contributing to the more efficient management and optimal allocation of resources.

Several promising areas for future research include the following:

(1) The development of models for quantifying cluster capital. It is necessary to continue the research using network analysis and machine learning. This will help to more accurately predict the dynamics of the capital within clusters. The addition of socio-economic regressors, such as the economic environment and clustering potential [66], can improve the accuracy of the calculations. Models that consider the multiplicative value of the total capital will allow for a more effective assessment of the impacts of different capital types. This will maintain the sustainability and innovation activity of the cluster;

(2) The development of intra-cluster connections. Deepening cooperation between cluster members and external stakeholders can help strengthen integration ties and increase economic efficiency. Economic models consider the leverage of clusters and separate cost accounting. Developing these models will allow for more accurate predictions of the outcomes within clusters;

(3) The study of mechanisms for capital co-evolution. A deeper analysis of the interactions between technological, intellectual, and financial capitals from the perspective of complementarities will help develop new methods of cluster integration and effective strategies for capital management. We can contribute to a systematic approach to cluster strategy by using multidimensional matrix methods, similar to those proposed by Kleiner and Kvint. This enhances its integration across different levels.

Thus, the results of this study contribute to the development of the theory of cluster system management. They emphasize the importance of integrating various types of capitals and optimizing their distribution to create innovative ecosystems. These findings can be useful for government agencies and business communities interested in improving the management of cluster structures.

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