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ON DIFFERENTIATION OF SOME GENERALIZED BANACH-SPACE-VALUED INTEGRALS

Here we study whether some results on differentiation of generalized integrals known in the scalar-valued case remain valid for Banach-spacevalued case and consider application to some problems in harmonic analysis on compact abelian zero-dimensional groups which depend on or can be reduced to the corresponding problems in the theory of differentiation or integration of Banach-valued functions on such a group.

We define a derivation basis on a group G with the above mentioned properties, introduce a notion of derivative (strong and weak) of a Banach-valued function with respect to this basis and show that a Henstock-Bochner type integral (or Henstock-Pettis type in the weak case) solves the problem of recovering a differentiable function from its respective derivative for any Banach space. In the opposite direction, the statement on differentiation of the indefinite Henstock type integral is true only for Banach spaces of finite dimension. Namely we prove that for any infinite-dimensional Banach space there exists a function on a group G, with values in this space, integrable on G in the sense of the Henstock-Bochner-type integral with the indefinite integral being nowhere differentiable with respect to the derivation basis introduced on the group.

A relation between convergence properties of a series with respect to the system of characters of the group G and differential properties of a certain function associated with the series is established and in this way the problem of recovering, by generalized Fourier formulae, the Banachspace-valued coefficients of a convergent series is solved for any Banach space by reducing it to the above mentioned problem of recovering a function from its derivative using the considered Henstock-type integral. Some results related to the convergence of Fourier-Henstock series with respect to characters are obtained. In particular it follows that for any infinite-dimensional Banach space there exists a function with values in this space such that its Fourier-Henstock series diverges everywhere on G. It is also shown that the rate of growth of the partial sums of such a divergent series depends on a structure of the considered Banach space.

Some of the presented here results are a generalization of those ones obtained in [1] and [2].

References

- [1] Dilworth, S. J., Girardi Maria, Nowhere weak differentiability of the Pettis integral, Quaest. Math. 18 (1995) no. 4, 365–380.
- [2] Skvortsov, V. A., Integration of Banach-valued functions and Haar series with Banach-valued coefficients, Moscow Univ. Math. Bull. 72 (2017) no. 1, 24–30.