Atlas of Aftershock Sequences of Strong Earthquakes

A. V. Guglielmi, O. D. Zotov and A. D. Zavyalov

Abstract This year marks the 150th birthday anniversary of the outstanding seismologist Fusakichi Omori. Our paper is devoted to this significant event. We found that the well-known Omori law may be represented in the form of a differential equation describing the evolution of aftershocks. This allows us to formulate the inverse problem of physics of the earthquake source which is "cooling down" after the main shock. The paper gives the examples of solving the inverse problem and illustrates a possibility to create the atlas of aftershocks after a series of strong earthquakes. The atlas contains a description of the parameters, the original sequence of aftershocks, and the so-called deactivation function for each event. The analysis of the atlas showed a rich variety of the evolution forms of the earthquake source after the main shock.

Keywords Seismology · Omori law · Aftershocks equation · Deactivation function · Earthquake source · Inverse problem · Phase portrait

1 Introduction

Our work is devoted to the memory of the outstanding Japanese scientist Fusakichi Omori (1968–1923). In 1896, he discovered a hyperbolic dependence of the frequency of aftershocks on time:

$$n(t) = k/(c + t). \tag{1}$$

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T. B. Yanovskaya et al. (eds.), *Problems of Geocosmos—2018*, Springer Proceedings in Earth and Environmental Sciences, https://doi.org/10.1007/978-3-030-21788-4_15

And this was the first empirical law of the physics of earthquakes [1]. Here the parameter k is the most important integral characteristic of the earthquake source that "cools" after the main shock. The parameter c is of no interest to us in the context of this work. In essence, (1) is the one-parameter formula.

So, the Omori law is represented by a rather rigid one-parameter formula. Many authors have worked to make the Omori law more flexible. The idea of R. Hirano and T. Utsu to replace the denominator in Formula (1) with the expression $(c + t)^p$ was widely spread in the literature [2, 3]. Unlike the Omori formula, the Hirano-Utsu formula is two-parameter.

We have chosen a different path, and our idea is this. On the one hand, we do not want to introduce additional phenomenological parameters that do not have a geodynamic meaning. That is, we want to leave the Omori formula with one parameter. On the other hand, we would like to take into account the nonstationarity of rocks in the earthquake source after the main shock. In other words, we must bear in mind that the value of k depends on the time, thereby reflecting the nonstationary relaxation of the of the earthquake source to a new metastable state.

However, it is impossible to replace k with $k(t)$ in Formula (1). We will do otherwise. First, we make the following substitution $k = 1/\sigma$. This is just a redefinition. We will call σ the deactivation coefficient. The value of σ tells us how fast the earthquake source loses its ability to excite the aftershocks. Now we use the evolution equation of aftershocks

$$dn/dt + \sigma n^2 = 0, \tag{2}$$

which is completely equivalent to the Omori law (1), if sigma does not change over time. The advantage of the law in the form of a differential equation (2) is that now nothing prevents us from taking into account the nonstationarity of the geological environment after the main shock. We solve the equation and obtain a generalization of the law of aftershocks evolution:

$$n(t) = n_0 \left[1 + n_0 \int_0^t \sigma\left(t'\right) dt' \right]^{-1} \tag{3}$$

Our version (3) preserves the hyperbolic structure of the Omori law, but it takes into account that time in the earthquake source, figuratively speaking, flows unevenly [4]. If $\sigma = \text{const}$, then in this and only in this case the generalized Formula (3) coincides with the classical Omori Formula (1) up to notation.

2 Inverse Problem of the Aftershocks Physics

We show that the generalized law of evolution (3) is useful for processing and analyzing of the aftershock sequences. Let us rewrite (3) in the form

$$\int_0^t \sigma(t')dt' = \frac{1}{n(t)} - \frac{1}{n_0} \equiv g(t). \tag{4}$$

Here $g(t)$ is an auxiliary function. It is known from experiment. This mathematical expression resembles the Volterra integral equation of the first type (with trivial kernel). This circumstance prompted us the idea to set and solve the inverse problem, i.e. to find the unknown deactivation factor σ by using the known frequency of aftershocks n.

Like almost every inverse problem, our inverse problem is incorrectly posed. Regularization consists in smoothing the auxiliary function: $g(t) \to \langle g(t)\rangle$. Solution of the problem is of the form

$$\sigma(t) = d\langle g(t)\rangle/dt. \tag{5}$$

3 Atlas of Aftershocks

According to the results of solving the inverse problem, we plan to release an "Atlas of Aftershocks" (ATAS). Currently, about a dozen solutions have been accumulated and in this section we want to show the first results. Let us say that we conceived ATAS as a collective project.

Figure 1 gives an idea about the content of the atlas [5]. Here are three events that occurred in California according to the catalogs http://www.data.scec.org and http://www.ncedc.org. Aftershock frequencies $n(t)$ are shown in the upper rows. Here are the dates and magnitudes of the main shocks. The middle panels show auxiliary functions $g(t)$ before and after regularization. The bottom panels give an idea of the variety of forms of the deactivation function $\sigma(t)$. We observe growing, decreasing and oscillating fragments of $\sigma(t)$. Let us pay special attention to the so-called Omori epoch, i.e. for long time intervals, when the value of σ is constant and, thus, the Omori law (1) is strictly satisfied. The duration of Omori epoch varies from case to case in a wide range. According to the presence or absence of the Omori epoch and other characteristic properties of the function $\sigma(t)$ one can try to make a classification of earthquake sources in the course of further work.

One more interesting event is shown in Fig. 2. In Southern California at a close (~1 km) distance from each other, three rather strong earthquakes occurred: the first with M = 5.4 at 22 h 39 m 59 s 1995.08.17, the second with M = 5.8 after 34 days

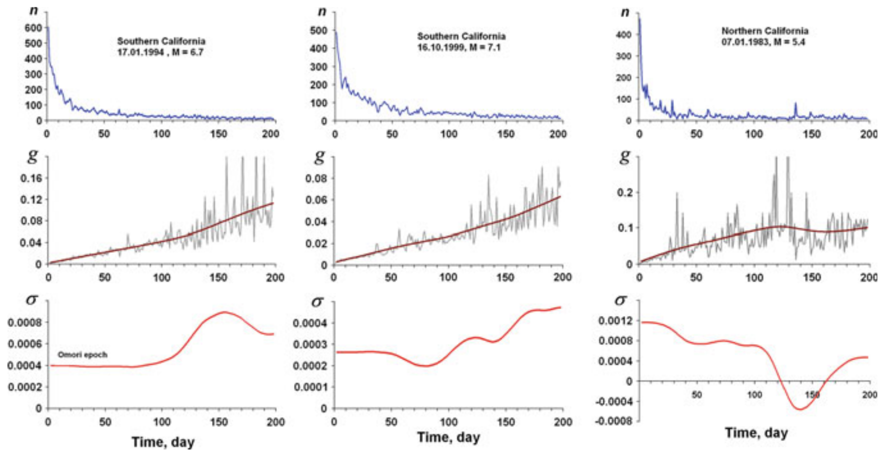


Fig. 1 The first three sheets of an aftershocks atlas (please, see the text)

at a distance of about 1 km, the third with M = 5.8 after 120 days at a distance of about 1 km from the second.

In the framework of the ATAS project, we want to consider the earthquake source as a dynamic system. It is known that the construction of phase portraits is an effective means in the study of dynamical systems. There is an idea to represent the evolution of aftershocks in the form of a trajectory in the phase planes. Examples of phase portraits of the earthquake source are presented in Fig. 3.

Here we used the data on earthquakes presented in Fig. 2. The top panel in Fig. 3 gives an idea of the portrait in the phase plane (g, σ). The point on the phase trajectory shows the initial state of the dynamical system, and the arrow indicates the direction of the system's movement along the trajectory. In the bottom panel we see the phase portrait in the plane $(\sigma, \dot{\sigma})$, where $\dot{\sigma} = d\sigma/dt$. We hope that the phase portraits of such sort will make it possible to produce a specific classification of the earthquake sources.

4 Discussion

It is clear that observations are the main source of our knowledge, and we also understand that the selection of empirical formulas plays a key role in systematizing our experience. Sometimes the empirical formula turns out to be fundamental.

Omori proposed the one-parameter formula to describe the flow of aftershocks. However, another empirical formula, namely, the two-parameter Hirano-Utsu formula, has become widely spread in seismology. The simple and elegant Omori Formula (1) was forgotten, its hidden possibilities were not realized and were not used.

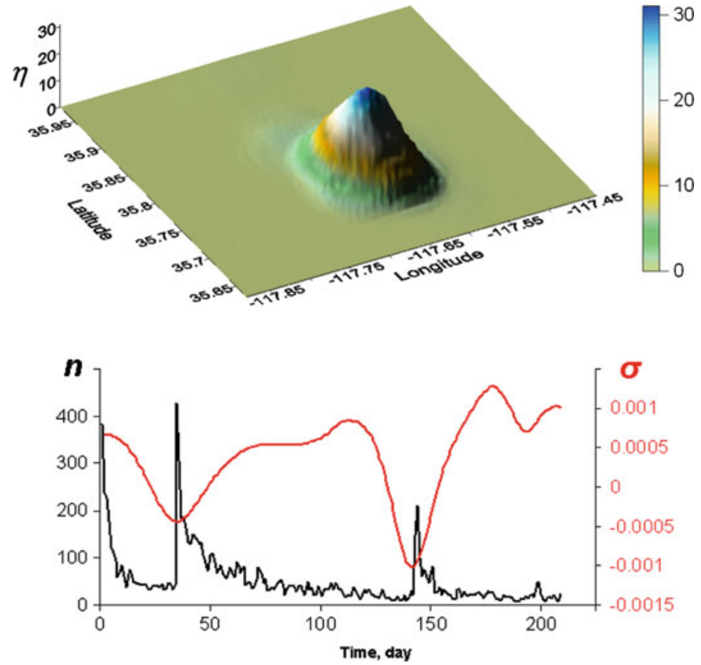
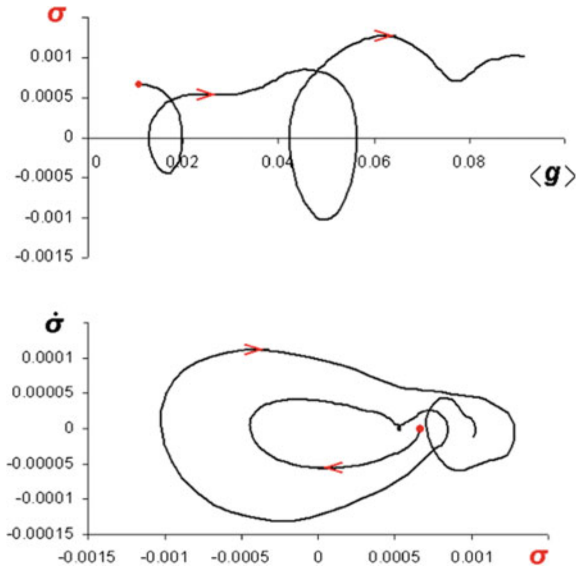


Fig. 2 Earthquakes in Southern California. Top panel shows the distribution of the aftershocks number on the earth's surface. The bottom panel shows three strong earthquakes and their aftershock sequences in the epicentral zone with a radius of 0.2° from August 17, 1995 till March 14, 1996 (black line) and the corresponding deactivation function σ(t) (red line)

Fig. 3 Phase portraits of the earthquake source in the planes (g, σ) and $(\sigma, \dot{\sigma})$ (top and bottom panels correspondingly)



The situation changed after we replaced Formula (1) with differential equation (2). The prospect of generalization (3) was opened immediately.

The differential equation of evolution (2) suggests also other interesting generalizations. For example, we can add a diffusion term to the right-hand side of (2) to take into account the space-time distribution of aftershocks:

$$\partial n/\partial t + \sigma n^2 = \hat{D}\nabla^2 n. \tag{6}$$

Here $\hat{D}$, generally speaking, should be considered as a tensor $\hat{D} = \mathrm{diag}\,(D_\parallel, D_\perp)$ in order to take into account the anisotropy of faults in the earth's crust, $D_\parallel \gg D_\perp$.

5 Conclusion

The well-known Omori law (1) we presented in the form of differential equation (2) describing the evolution of aftershocks. The evolution equation of aftershocks gave us the opportunity to set the inverse problem of physics of the earthquake source, which is "cooling down" after the main shock. The essence of the inverse problem consists in determining the deactivation function $\sigma(t)$ from the frequency of aftershocks $n(t)$ known from observation. We presented a concrete example of solving the inverse problem. There is no doubt that the use of our aftershock processing technique opens up new experimental possibilities.

We are planning to create an atlas of aftershocks after the strong main shocks. The atlas will contain a description of the basic parameters, the initial sequence of aftershocks, the function of deactivating the source $\sigma(t)$ and the phase portrait for each event. A preliminary analysis revealed the exceptional wealth of the evolutionary forms of earthquake source after the main shock.

The authors thank the staff of Northern California Earthquake Data Center (NCEDC) and Southern California Earthquake Center (SCEC) for providing the catalogues of earthquakes. This research was partially supported by the Russian Foundation for Basic Research (Grant No. 18-05-00096), the Program 28 of the Presidium of RAS, and Russian Governmental assignment Programs 0144-2014-0097, 0144-2014-00103. There are no potential conflicts of interest to disclose.

References

1. Omori, F.: On the aftershocks of earthquake. J. Coll. Sci. Imp. Univ. Tokyo **7**, 111–200 (1894)
2. Hirano, R.: Investigation of aftershocks of the great Kanto earthquake at Kumagaya. Kishoshushi, Ser. 2 **2**, 77–83 (1924)
3. Utsu, T.: A statistical study on the occurrence of aftershocks. Geophys. Mag. **30**, 521–605 (1961)
4. Guglielmi, A.V.: Omori law: a note on the history of geophysics. Phys. Usp. **60**, 319–324 (2017)
5. Guglielmi, A.V., Zavyalov, A.D.: Omori law: to the 150th anniversary of Fusakichi Omori. J. Volcanol. Seismol. **12**(5), 353–358 (2018)