BOUNDARY CONDITIONS FOR THE USADEL EQUATIONS AND PROPERTIES OF DIRTY S-N-S SANDWICHES

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Résumé.- Il est démontré que les conditions aux limites pour les équations d'Usadel dans le cas d'une limite plane consistent en la continuité de F et ND $(\vec{\nabla} - i2e\vec{A}/\hbar)F$. Ces conditions sont utilisées pour l'obtention des conditions aux limites effectives à l'interface S-N pour les équations de Guinzbourg-Landau, aussi bien que pour l'obtention de la relation courant-phase d'une jonction Josephson S-N-S, ayant un faible libre parcours moyen.

Abstract.- Boundary conditions for the Usadel equations at a plane wall conducting interface are found to consist in continuity of F and ND $(\vec{\nabla} - i2e\vec{A}/\hbar)F$. These conditions are used to find the effective boundary conditions for the Ginzburg-Landau equations at the S-N interface, as well as the current-phase relationship for S-N-S sandwiches with a short electron mean free path.

INTRODUCTION.- The Usadel equations/1/ give one a convenient tool for calculating the dc properties of superconducting structures in the dirty limit. However, it was still unclear which boundary conditions are to be used for these equations in the analysis of the properties of S-N and S-N-S structures.

In this paper the proper boundary conditions as well as the results of their application to two basic problems are reported.

BOUNDARY CONDITIONS.- Starting from the Eilenberger equations/2/ one can show that within the dirty limit these are the Usadel functions $F(r,\omega)$ which are to be continuous at the plane(*) interface between two metals in the case of small electron reflection from the boundary. Taking into account the conservation of current, we find that the product $ND(\vec{\nabla} - i2e\vec{A}/\hbar c)F$ must also be continuous. Here N is the Fermi surface density of states, $\vec{A}$ is the vector potential, $D = v_F\ell/3$ is the diffusion coefficient, and ℓ is the electron mean free path which is assumed to be much less than the characteristic coherence length $\xi = (\hbar D/2\pi kT)^{1/2}$.

The Usadel equations, together with the boundary conditions given above, are valid at arbitrary temperatures. However, in this work we restrict ourselves to their application to the situations where the normal metal with a zero critical temperature contacts the superconductor with $T_c \approx T$. The Ginzburg-Landau (GL) equations can be used in this case only at distances much larger than ξ from the interface, and a more general theory is to be used to obtain the effective boundary conditions for the GL order parameter $\Delta(\vec{r})$/3/. Using the Usadel equations we can get these conditions in a much easier way than with the help of more complex integral equations discussed by Zaitsev/4/.

(*) The products NF and $D(\vec{\nabla} - i2e\vec{A}/\hbar c)F$ are to be continuous at the interface with the diffuse electron scattering.

S-N INTERFACE.- If the thickness d of the normal layer is large ($d \gg \xi_N$), the supercurrent through the boundary is absent, Δ can be taken to be real, and the effective boundary condition for the GL equations has the form/3/

$$\vec{n}\vec{\nabla}\Delta/\Delta = b^{-1} \qquad (1)$$

The parameter b appears to be dependent on the following relation

$$\gamma = N_N D_N^{1/2}/N_S D_S^{1/2} = (\sigma_N/\xi_N) \;/\; (\sigma_S/\xi_S) \qquad (2)$$

At $\gamma \ll 1$ the result obtained by Zaitsev/4/

$$\xi_s/b\gamma = \xi(T)/b\Gamma = \sum_{n=0}^{\infty}(2n+1)^{-3/2}/\sum_{n=0}^{\infty}(2n+1)^{-2} \approx 1.369 \qquad (3)$$

holds, where $\xi(T) = [\pi\hbar D_s/8k(T_c-T)]^{1/2}$ is the coherence length of the superconductor, and the parameter

$$\Gamma = \gamma(\xi(T)/\xi_s) = (\pi/2)\gamma[T/(T_c-T)]^{1/2} \qquad (4)$$

can be of the order of unity.

However, if γ is about or more than unity, the result (3) is wrong. At $\gamma \gg 1$ we get the other simple expression

$$\xi(T)/b\Gamma = \sum_{n=0}^{\infty}(2n+1)^{-2}/\sum_{n=0}^{\infty}(2n+1)^{-5/2} \approx 1.117 \qquad (5)$$

but for $\gamma \sim 1$ the Usadel equations are to be solved numerically. Figure 1 shows the result of the calculation.

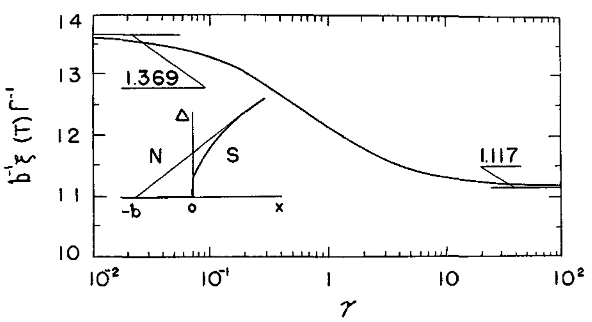

Fig. 1 : Dependence of the effective length b (see insert), determining the effective boundary condition (1) for the Ginzburg-Landau equation, on the parameter γ (2) for the plane S-N interface.

S-N-S SANDWICH.- Analysing the Josephson effect in dirty ($\ell \ll \xi_N, d$) sandwiches, the most interesting situation is encountered when ξ_s and ξ_N are comparable, but the conductivity σ_N of the interlayer material is much less than that of the electrodes (σ_s). For this case, the parameter γ can be assumed to be small, and instead of equation (3) we get for the parameters $p = \Gamma^{-1}\xi(T)\vec{n}\vec{\nabla}\mathrm{Re}\Delta/\mathrm{Re}\Delta$ and $q = \Gamma^{-1}\xi(T)\vec{n}\vec{\nabla}\mathrm{Im}\Delta/\mathrm{Im}\Delta$ at the interfaces ($x = \pm d/2$):

$$p = \pm\sum_{n=0}^{\infty}(2n+1)^{-3/2}\tanh\left[(2n+1)^{1/2}d/2\xi_N\right]/\sum_{n=0}^{\infty}(2n+1)^{-2} \tag{6}$$

$$q = \sum_{n=0}^{\infty}(2n+1)^{-3/2}\coth\left[(2n+1)^{1/2}d/2\xi_N\right]/\sum_{n=0}^{\infty}(2n+1)^{-2}$$

The gauge is chosen to that $\vec{A} = 0$ and $\mathrm{Im}\Delta(x=0) = 0$. The results of solution of the GL equations depend on two dimentionless parameters : Γ and d/ξ_N.

For $\Gamma \ll \min[1, d/\xi_N]$, the values of $|\Delta|$ at the interfaces are close to its equilibrium value Δ_o. Thus, the relationship between the supercurrent I_s and the phase difference ϕ of the order parameter at the interfaces is just the same as in the S-N-S variable thickness bridges (VTB)/5/ :

$$I_s/I_o = \frac{d(q-p)}{2\xi_N} \sin\phi \tag{7}$$

where $I_o = \pi\Delta_o^2/4ekTR_N \sim (T_c - T)$ is the maximum possible value of the critical current in the Josephson structure with the normal resistance R_N.

At large values of Γ ($\Gamma \gg \max[1, \xi_N/d]$) the order parameter at the boundaries of the interlayer is reduced because of the proximity effect. If the thickness d is much larger than ξ_N, this suppression of $|\Delta|$ is independent of ϕ and the $I_s(\phi)$ relationship is sinusoidal. On the contrary, at $d/\xi_N \lesssim 1$ the superconductors "feel" the phase of each other through the interlayer, so the suppression is strongest at $\phi \simeq \pi$ and weakest at $\phi \simeq 0$. Since I_s is proportional to $|\Delta|^2$, the dependence $I_s(\phi)$ deviates from the sinusoidal form and has a maximum at $\phi < \pi/2$

$$I_s/I_o = \frac{d}{2\xi_N\Gamma^2}\,\frac{q-p}{p^2\cos^2(\phi/2)+q^2\sin^2(\phi/2)}\sin\phi$$

$$I_c \sim (T_c - T)^2 \tag{8}$$

If the interlayer is thin ($d/\xi_N \ll \Gamma, \Gamma^{-1}$), the supercurrent through the sandwich can be strong enough to cause the depairing effect in the electrodes. The corresponding critical current

$$I_c/I_o = \frac{2}{3\sqrt{3}}\,\frac{d}{\xi_N\Gamma} \; ; \; I_c \sim (T_c - T)^{3/2} \tag{9}$$

is also achieved when $\phi < \pi/2$.

Figure 2 shows the transition from the VTB limit (7) to the depairing limit (9) with decreasing sandwich thickness at the fixed value of Γ. These

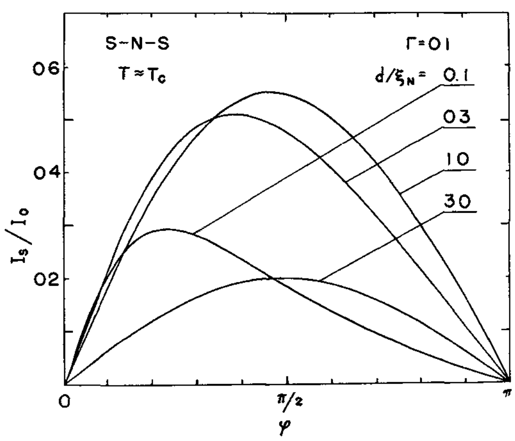

Fig. 2 : Current-phase relationships for the dirty ($\ell \ll d,\xi$) S-N-S sandwiches at $T \approx T_c$ for fixed materials of electrodes and interlayer ($\Gamma = (\sigma_N\xi(T)/\sigma_s\xi_N) = 0.1$) and various values of the interlayer thickness d, $\xi_N = (\hbar D_N/2\pi kT)^{1/2}$, $I_c = \pi\Delta_o^2/4ekTR_N$.

plots, as well as the asymptotic expressions (7)-(9), demonstrate that for the given materials of the electrodes and interlayer the product I_cR_N has a maximum possible value at the intermediate thickness of the sandwich, close to the decay length ξ_N of the normal material of the interlayer.

References

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